

3D DENDRITE SHAPE IN THE LARGE CHEMICAL PÉCLET NUMBER LIMIT IN THE CASE OF ROTATIONAL SYMMETRY

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The solution of nonlinear integrodifferential equation for the mass transport defines the shape of dendrite, growing in a supersaturated alloy. We obtained a solution of this equation in the limit of large chemical Péclet number in the case when dendrite shape is a surface of revolution.

The equation (1) describes the shape of the growing dendrite as a function of supersaturation in steady-state non-equilibrium conditions [1]:

$$\frac{Q}{m_v c_p} \left[\frac{d_c}{\rho} K + \beta V \right] - C_{l\infty} = I_{\zeta}^{CH}, \quad (1)$$

$$I_{\zeta}^{CH} = \frac{2(1-k_v(V))}{\sqrt{1-P_c\tau_*}} \left(\frac{P_c}{2\pi} \right)^{3/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K_{1/2} \left(\frac{P_c \sqrt{b_H}}{1-P_c\tau_*} \right) \frac{C_i(\mathbf{x}_1)}{b_H^{1/4}} \exp \left(-\frac{P_c(\zeta(\mathbf{x}) - \zeta(\mathbf{x}_1))}{1-P_c\tau_*} \right) d^2 x_1 \quad (2)$$

where ζ is a moving phase transition interface, V - constant velocity of growth,

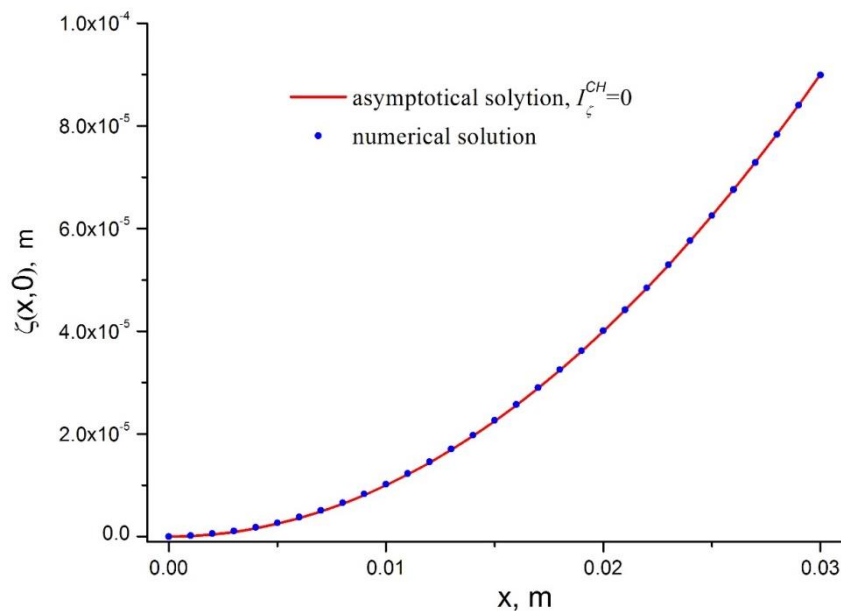
$P_c = \frac{V}{2D} \left(\frac{V}{V_D} \right)^2 \frac{1}{\tau_*}$ - chemical Péclet number, V_D - diffusion speed, C_i - in-

terfacial concentration, $b_H = (1-P_c\tau_*)|\mathbf{x} - \mathbf{x}_1|^2 + (\zeta(\mathbf{x}) - \zeta(\mathbf{x}_1))^2$ and K is an interfacial curvature. Retaining in the Bessel function only the leading term we get the two-dimensional Laplace-type integral that can be calculated with a saddle-point technique:

$$I_{\zeta}^{CH} = \frac{(1-k_v(V))C_i(x_0, y_0)(1-P_c\tau_*) \exp \left(\frac{P_c S(x_0, y_0)}{1-P_c\tau_*} \right)}{\sqrt{\left(\zeta_{x_1 x_1}'' S(x_0, y_0) + (1-P_c\tau_*) \right) \left(\zeta_{y_1 y_1}'' S(x_0, y_0) + (1-P_c\tau_*) \right) - \left(\zeta_{x_1 y_1}'' S(x_0, y_0) \right)^2}} \quad (3)$$

where $S(x_1, y_1) = \sqrt{b_H} + \zeta(x, y) - \zeta(x_1, y_1)$ and x_0 is a maximum point of S . Under assumption that interface function is a surface of revolution, we can consider only the section $y=0$. Then eq. (3) coincides with previously obtained equation for the 2D shape [2]. Expand the function ζ in a Taylor series in a vicinity of x_0 we realized that the integral contribution I_{ζ}^{CH} vanishes.

Asymptotically obtained interfacial function ζ is a sphere. The numerical solution of equation (3) is also close to a sphere, as shown in Figure 1. This result is in a agreement with limiting case of the steady-state solidification conditions with high Péclet numbers [1].



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1. Galenko P.K, Alexandrov D.V. and Titova E.A., Phil. Trans. R. Soc. A, (2017).
2. 2.Titova E. A. AIP Conference Proceedings 2015, 020102 (2018); doi: 10.1063/1.5055175